On DSS Method for a Fast Identification of the Static and Dynamic Responses of Servovalves*

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Abstract

In this work we consider a class of quasilinear systems of differential equations which allows to describe dynamics of electrohydraulic servovalves. A method for fast identification of static and dynamic responses, by a short-time experiment, is described.

1 Introduction

In 1950 Bill Moog invented a very reliable device to control flow in hydraulic systems by electric signals. Since that time electrohydraulic servovalves are key elements of various automatic control systems. Now it is impossible to imagine rockets, aircrafts, satellites, certain machine tools and control systems without

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servovalves. Such automatic systems require high performance of the servovalve. That is why research and development of such control systems as well as their maintenance require to test the dynamic and static parameters of servovalves. Unfortunately up to now, test equipment for servovalves has numerous of lacks. First of all, the test equipment needs powerful hydraulic pump stations, which require water cooling, noise insolation and frequent maintenance.[1] Moreover, this equipment requires large space and tubing for the distribution of the hydraulic fluid. The installation of such test equipment takes time and moving it is difficult. We must notice, that at the present time the complete test equipment costs more than $150,000 and a test of a servovalve takes about 15 minutes. The cost may be increased many times if to take into account that the different servovalves use different kinds of hydraulic fluids (yellow oil, red oil, kerosene, skydrol, water and so on). Notice that noise from the pump stations creates an additional problem for hydraulic test equipment. The point is that the presence of noise superimposes restrictions on the construction of buildings and the laboratories where the test stand is located. Today, we know how to make a quiet, compact, mobile and cheap hydraulic test stand.[2]

In the present paper an approach for testing dynamic and static characteristics of servovalves and principles of construction of modern test equipment are described. The basic idea of the new approach is to divide the test process into many test sub-processes, each of which takes a very short interval of time. This idea implies the possibility to use the energy of a hydraulic accumulator to produce a short subtest and, therefore, to make obsolete powerful pump stations. Using this idea, we have developed a series of mathematical methods, wrote computer programs and made computer simulations for various servovalves. The experiments showed a possibility to identify the static characteristic of a servovalve for 1 to 2 seconds of the test time. To identify the dynamic characteristics we use a number of sub-tests each of which requires just some milliseconds of the test time. So, using a 4kW motor to fill a three-liter accumulator, the complete test of a servovalve takes less than minute. It is necessary to note that the new approach is based on the construction of a dynamic model of the servovalve which is under test and this fact turns out to be extremely useful. The knowledge of the dynamical system opens unlimited possibilities for designers and developers of control systems. They can use Mathematica, MathCad, MatLab, Simulink, LabView and other software for fast simulations on a computer as well as for complex mathematical analysis.

2 Statement of the problem

A servovalve is a complex dynamical system and the complete description of all its parameters, obviously, is not possible. However, most of engineers and developers are interested in some basic dynamic and static characteristics. To describe dynamics of servovalves, at the present time, people usually consider

[1]The power of pump stations reaches 100 kW and more.
[2]The estimated expenses for the equipment are less $25,000.

amplitude-phase frequency responses. In this case, obviously, it is assumed implicitly that dynamics of a servovalve can be described well by a linear differential equation or by a linear transfer function.[3] In practice, to describe the dynamic parameters, engineers usually use the following system of differential equations of the second order (see for instance [1], [2], [3]).

$$A\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Cx = u(t), \quad \frac{dx}{dt}(0) = x(0) = 0. \tag{1}$$

Here, $u(t)$ is an input electrical signal, $x(t)$ is the output flow of a liquid, A, B and C are some fixed parameters of the servovalve.

The static characteristic $x = F(u)$ of the servovalve is close to the linear function $x = u/C$, however, in practice, there is the necessity to know the static behaviour much better. In this case, by definition, to construct the static response, it is necessary to know the output flow x at any fixed control u [4]. Here, of course, it is supposed, that at fixed control the servovalve is a structurally stable dynamic system with a unique stable equilibrium point. The such direct construction of the static characteristic, obviously, requires time because it is necessary to check the valve at all the points of the control range and each of the points requires a delay for the transition time. In practice, to find the statics approximately, people usually construct the Lissajous figure $(u(t), x(t))$, where $u(t)$ is a very low-frequency harmonic control signal (usually less 0.01Hz). But in any case it takes time.

In the present work, to take into account the nonlinearity of the static characteristic, we shall consider the following quasilinear dynamic model of a servovalve.

$$A\frac{d^2x}{dt^2} + B\frac{dx}{dt} + f(x) = u(t), \quad \frac{dx}{dt}(0) = x(0) = 0. \tag{2}$$

This system differs from the system (1) by the presence of a non-linear function $f(x)$. In general, such a nonlinearity can lead to the appearance of a very complex chaotic behavior.[4] However, a real servovalve is a stable dynamical system and its static response is close to a linear function. Therefore we shall require that the function $f(x)$ is a monotone one and is close to the function Cx. Moreover we shall assume, that the roots of the characteristic equation $A\lambda^2 + B\lambda + C = 0$ of the corresponded linear system have negative real parts. These conditions guarantee a stability of the system (2) at any constant control $u(t) = const$ and, therefore, the static characteristic $x = F(u)$ of the servovalve is the non-linear function $x = f^{-1}(u)$. The experiments made by the authors of the present work have shown that the system (2) describes well both static and dynamic characteristics of servovalves. Therefore the main problem to test a servovalve is reduced to the identification of the parameters A, B and the function $f(x)$ of the system (2). To solve this problem it is necessary to suggest a form of the test signal $u(t)$ and give an algorithm for a numerical analysis

[3] In a non-linear case the amplitude-phase response depends on amplitude and form of the input signal and, therefore, does not make big sense.

[4] In 1918 Duffing found a chaos in a similar system with a cubic nonlinearity. See, for instance, [5].

of $x(t)$ which identifies A, B and $f(x)$. Notice that we are interested in such methods which give a good solution of the problem for a short enough time of testing. Moreover, the methods should be structurally stable to the presence of noise and measurement errors.

3 Amplitude-phase frequency responses of a quasilinear system

Let a solution $x(t)$ of the system (2) correspond to the input harmonic signal $u(t) = \sin(\omega t)$. After some transition time Δt the output signal $x(t)$ will be a periodic function with the same frequency ω. In this case, obviously, $x(t)$ can be represented by the following Fourier series expansion:

$$x(t) = \sum_{k=0}^{\infty} R_k(\omega)\sin(k\omega t + \varphi_k(\omega)).$$

For any k, the amplitude $R_k(\omega)$ and the initial phase $\varphi_k(\omega)$ of the corresponding harmonic are calculated by the formulas:

$$R_k(\omega) = |K_k(i\omega)|\,, \quad \varphi_k(\omega) = \arg\left(K_k(i\omega)\right), \qquad (3)$$
$$K_k(i\omega) = \frac{\omega}{2\pi}\int\limits_{\Delta t}^{\Delta t + 2\pi/\omega} x(t)e^{-ik\omega t}dt.$$

Let us note that the amplitude $R_1(\omega)$ and the phase lag $\varphi_1(\omega)$ of the first harmonic are known as the amplitude-phase characteristics of the dynamic system. Namely these functions represent the most of interest at the research and the development of automatical control systems.

There are many different methods to construct the frequency responses $R_1(\omega)$, $\varphi_1(\omega)$. The most simple method is to use the formulas (3) to calculate the functions directly. In this case, to construct the frequency response, it is necessary to make measurements for a larger series of frequencies $\omega = \omega_1, \dots, \omega_N$, (usually $N > 20$) and, then, to find the corresponding interpolation curves $R_1(\omega)$, $\varphi_1(\omega)$.

The other method is to identify the parameters A, B and function $f(x)$ of the quasilinear system (2). In this case the test function $u(t)$ can be chosen from a wide class of functions (for example we can use the step signal $u(t) = 1(t)$). In order to identify A, B and $f(x)$ the following optimisation problem, for instance, can be solved:

$$\min_{A,B,f} \sum_{k=1}^{N} \left(x(t_k) - x_{A,B,f}(t_k)\right)^2. \qquad (4)$$

Here $x(t_k)$ are the results of measurement (samples) of the output signal and $x_{A,B,f}(t_k)$ is a solution of the quasilinear system (2) with parameters A, B, f for the control function $u(t)$ at the moments $t_k \in (\tau_1, \tau_2)$. The minimization is performed with respect to A, B and f, where f is a function of a finite-parametric family of functions f_μ. So, the minimization is performed with respect to parameters A, B and μ. The knowledge of parameters A, B and f, obviously, allows to construct the frequency responses. This method allows to identify the parameters of the system for an arbitrarily small time interval of measurement (τ_1, τ_2). However, the method requires a high precision in measurement of the output signal $x(t_k)$.

In our case to find the frequency responses $R_1(\omega)$, $\varphi_1(\omega)$ of servovalves it is expedient to use a method for construction of dynamic characteristics which takes into account the proximity of the system (2) to the linear one (1). In this case, it is natural to suppose, that $R_1(\omega)$, $\varphi_1(\omega)$ are very similar to the frequency responses of the linear system (1) and our problem is reduced to search the optimum parameters A^*, B^* and C^*. Of course, A^*, B^* and C^* can be found as a solution of (4) but we suggest an other way. It is well known, that the frequency responses of the linear system (1) can be computed by the following formulas:

$$\begin{aligned} &R(\omega) = |K(i\omega)|\,, \quad \varphi(\omega) = \arg(K(i\omega)), \\ &K(s) = 1 \big/ \left(As^2 + Bs + C\right). \end{aligned} \tag{5}$$

Here $K(s)$ is the transfer function of the system (1). In order to find parameters A^*, B^* and C^* we suggest to solve the following optimisation problem:

$$\min_{A,B,C} \sum_{k=1}^{N} \left|K(i\omega_k) - K_1(i\omega_k)\right|^2, \tag{6}$$

where $\omega_1, \ldots, \omega_N$ is a series of test frequencies and $K_1(i\omega_1), \ldots, K_1(i\omega_N)$ are values of the transfer function which were computed by (3). In contrast to the above mentioned methods the last one does not require high precision in measurement of the output signal $x(t_k)$ and gives good results fast enough because the method works for small series of experiments ($N \geq 2$).

4 The DSS method

In order to identify the static response we use the DSS method. The basic idea of this method is the following. Let $x(t)$ be an output signal of the dynamic system (2) for a test signal $u(t)$. Then, obviously, the following relation is fulfilled:

$$f(x(t)) = u(t) - A\frac{d^2x}{dt^2}(t) - B\frac{dx}{dt}(t). \tag{7}$$

That means that the vector-function

$$(x(t), f(x(t))) = \left(x(t), u(t) - A\frac{d^2x}{dt^2}(t) - B\frac{dx}{dt}(t)\right) \in \mathbb{R}^2 \tag{8}$$

is a parametric form of the static characteristic $u = f(x)$ (or $x = f^{-1}(u)$). We can consider that the term

$$- A\frac{d^2x}{dt^2}(t) - B\frac{dx}{dt}(t) \tag{9}$$

in (8) is a dynamical correction of the Lissajous figure $(x(t), u(t))$. Namely this term allows to construct the static characteristic of the system for a short time, i.e. without a delay for the relaxation of transition processes. However, to evaluate the dynamic correction (9), it is necessary to know parameters A and B of the system (2). Below we describe a method of successive approximations to find A, B and $f(x)$. First of all, we solve the problem (6) to find parameters A^*, B^* and C^*. The found parameters A^*, B^* and the linear function C^*x is the first approximation A_0, B_0 and $f_0(x)$ of the corresponding parameters A, B and function $f(x)$ of the system (2). To construct the next approximations A_{n+1}, B_{n+1} and $f_{n+1}(x)$ $(n = 0, 1, 2, \ldots)$ we use the following inductive rule. Let assume that the parameters A_n, B_n and function f_n are known and $x_n(t)$ is a solution of

$$A_n\frac{d^2x_n}{dt^2} + B_n\frac{dx_n}{dt} + f_n(x) = u(t), \quad \frac{dx}{dt}(0) = x(0) = 0. \tag{10}$$

Then:

1. Substituting the parameters A_n and B_n into the formula (7) instead of A and B, we find the next approximation $f_{n+1}(x)$ of the function $f(x)$.

2. The next generation A_{n+1} and B_{n+1} of the parameters A and B can be found as a solution of the following optimisation:

$$\min_{A_{n=1}, B_{n+1}} \sum_{k=1}^{N} \left(x(t_k) - x_{A_{n+1},B_{n+1},f_{n+1}}(t_k)\right)^2. \tag{11}$$

   Here $x(t_k)$ are the results of measurement (samples) of output signal and $x_{A_{n+1},B_{n+1},f_{n+1}}(t_k)$ is a solution of the quasilinear system (2) with parameters A_{n+1}, B_{n+1}, f_{n+1} for the control function $u(t)$ at the moments $t_k \in (\tau_1, \tau_2)$. Notice, the minimization is performed with respect to A_{n+1} and B_{n+1} only.

The described steps should be repeated until the acceptable approximation will be found. One more significance of our method is that, there are no big needs to compute the derivatives $\frac{dx}{dt}(t_k)$ and $\frac{d^2x}{dt^2}(t_k)$ numerically or to use for that special electronics. The point is that we can use the derivatives of the approximated solution $x_n(t) \equiv x_{A_n,B_n,f_n}(t)$ which was obtained on the n^{th} step of the iteration process. In this case the n^{th} approximation of the static characteristic is given by the following parametric form:

$$(x(t), f(x(t))) = \left(x(t), u(t) - A_n\frac{d^2x_n}{dt^2}(t) - B_n\frac{dx_n}{dt}(t)\right). \tag{12}$$

By (10) the formula (12) takes the following simple form:

$$(x(t), f(x(t))) = (x(t), f_n(x_n(t))).$$

So, to construct the static characteristic we do not need to compute the derivatives of the output signal $x(t)$. That fact is extremely important. The point is that the measured signal $x(t)$ contains noise and the measuring errors which lead to big problems at the differentiation.

In conclusion we must note that we did not investigate all questions about convergence of our DSS process however the computer simulations demonstrated fantastic results. Moreover, the acceptable results can be obtained even on the first step of the DSS process if to use a low frequency input signal $(1-2$ Hz).

Notice also that the DSS method can be very simply generalised onto a quasilinear systems of high order. For example, we tested our approach for the following system of differential equations of the third order:

$$A\frac{d^3x}{dt^3} + B\frac{d^2x}{dt^2} + C\frac{dx}{dt} + f(x) = u(t), \quad \frac{d^2x}{dt^2}(0) = \frac{dx}{dt}(0) = x(0) = 0.$$

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